Homogeneous Interference Game in Wireless Networks

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Abstract. We consider the problem of joint usage of a shared wireless channel in a an interference-bound environment, and focus on a distributed setting where there is no central entity managing the various transmissions. In such systems, unlike other multiple access environments, several transmissions may succeed simultaneously, depending on spatial interferences between the different stations. We use a game theoretic view to model the problem, where the stations are selfish agents aiming at maximizing their success probability. We show that when interferences are homogeneous, system performance suffers an exponential degradation in performance at an equilibrium, due to the selfishness of the stations. However, when using a proper penalization scheme for aggressive stations, we can ensure the system's performance value is at least $1/e$ of the optimal value, while still being at equilibrium.

1 Introduction

Wireless networks often involve the joint usage of common communication channels in a multiple access environment. In most of the models capturing such settings, simultaneous transmission by more than one station results in a collision causing all transmissions at that time to fail. CSMA-type methods are usually used in such scenarios in order to deal with collisions, in an attempt to maximize the system's throughput. However, in many current wireless networks, such as mesh WiFi networks, or 802.15 clusters, simultaneous usage of the same wireless channel is possible. Consider, for example, the settings described in Fig. 1, where we outline two stations, A, B and their transmission ranges. If the clients of A and B are a and b respectively, then simultaneous transmissions will cause a collision at client a, while b can receive the message from B. However, if the clients of A and B are a' and b respectively, then simultaneous transmissions will both succeed, since they do not collide at either of the receiving ends.

In wireless networks where channel access need not be exclusive, one of the major optimization issues is the efficient use of radio resources. In this paper we consider the problem of joint usage of a common communication channel by a finite number of stations, where stations are always backlogged, i.e., always have a packet to send. We present a generalization of classic multiple access models by introducing the notion of *spatial interference parameters*, which capture the pairwise interferences between the stations contending for the common radio resource. It is important to notice that in

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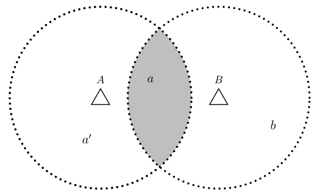

Fig. 1. Outline of two stations, A, B and their transmissions ranges.

this model several transmissions may succeed simultaneously, and thus the commonly assumed upper bound of one on the overall throughput of the system no longer holds. The overall number of successful transmissions at any time can take any value between 0 and n, where n is the number of stations in the network. The exact value depends on the inter-station interferences.

As a preliminary step in understanding this model, we focus our attention on the case of homogeneous interferences, where every station inflicts the same amount of interference on any other station. Indeed, such homogeneous interferences will usually not provide an accurate modeling for real-life scenarios where different agents have different interference patterns. Yet, we believe that a better understanding of the restricted settings (where the system is described by a single parameter) is both interesting and serves as an important step toward providing insights into understanding more general non-homogeneous environments.

Model. We model our problem as a game played by selfish agents. We consider a system consisting of n agents using a common wireless medium. For every agent i, we let $S = [0, 1]$ be the strategy space of agent i, and let $R_i \in S$ denote a strategy chosen by agent i. We refer to R_i as the *probability* that agent i transmits. Due to interferences, the probability of a *successful* transmission also depends upon the transmission of other agents. Given a profile $\overline{R} = (R_1, \ldots, R_n) \in [0, 1]^n$, we define the *success probability* of agent i's transmission as:

$$r_i(\overline{R}) = R_i \cdot \Pi_{j \neq i}(1 - \alpha_{i,j} R_j),$$

where for every $1 \leq i, j \leq n$, $\alpha_{i,j} \in [0, 1]$ is a fixed network-dependent parameter denoting the amount of interference inflicted on i upon simultaneous transmission of both i and j. One way of thinking about the $\alpha_{i,j}$-s is by viewing them as the probability that a transmission by j will interfere with a transmission of i. Given a profile $\overline{R} = (R_1, \ldots, R_n)$, the *social welfare* $\varphi(\overline{R})$ (also referred to as the *throughput* of the system) is considered to be the overall use of resources in the system, i.e.

$$\varphi(\overline{R}) = \sum_{i=1}^{n} r_i(\overline{R}) = \sum_{i=1}^{n} R_i \prod_{j \neq i}(1 - \alpha_{i,j} R_j).$$

$\varphi(\overline{R})$ can be interpreted as the expected number of successful transmissions.

Note that a-priori, $\varphi(\overline{R})$ can take any value between 0 and n. For example, if for all i, j, $\alpha_{i,j} = 0$, i.e., there are no interferences, then for the profile where $R_i = 1$ for all i

we obtain $\varphi(\overline{R}) = n$, which implies an optimal use of resources. On the other hand, if for all i, j, $\alpha_{i,j} = 1$, then the same profile obtains zero-throughput, i.e., $\varphi(\overline{R}) = 0$. In the latter case our model coincides with classic multiple access models, simultaneous transmissions result in a collision, causing all transmissions to fail.

We refer to the interferences as *homogeneous* if there exists some $\alpha \in (0, 1)$ such that for all i, j, $\alpha_{i,j} = \alpha$. By the above observations, when considering homogeneous interferences, we restrict our attention to the case where for all i, j, $\alpha_{i,j} = \alpha \in (0, 1)$. In what follows we refer to a profile $\overline{R}$ as *uniform*, if $R_i = R_j$ for all i, j.

For every agent i, we let $U_i(\overline{R})$ be the utility function of agent i, assuming agents play profile $\overline{R}$. In the following sections we consider several choices for these utility functions, and discuss the system's performance where agents are selfish, and aim at maximizing their own utility, regardless of the effect their choices have on the overall social welfare. We refer to the above setting as the *homogeneous interferences multiple-access (HIMA) game*.

Given any profile $\overline{R} = (R_1, \ldots, R_n)$, we let $\overline{R}_{-i}$ denote the subprofile defined by strategies of all agents except for agent i. We further let $(\overline{R}_{-i}, R'_i)$ be the profile where every agent other than i plays the same strategy as in $\overline{R}$, while agent i plays strategy R'_i. Profile $\overline{R}$ is said to be a *Nash equilibrium* (NE) if for every i, and every $R'_i \in [0, 1]$, $U_i(\overline{R}) \geq U_i(\overline{R}_{-i}, R'_i)$. Intuitively, a profile is an NE if no agent can increase its benefit by unilaterally deviating from his choice. We let $\overline{R}^{(n)}_{\mathrm{NE}}$ denote an NE profile for n agents, and use $\overline{R}^{(n)}_{\mathrm{OPT}}$ to denote any profile for n agents which maximizes the social welfare. Assuming an NE exists, we use the notion of *Price of Anarchy (PoA)* [1] in order to evaluate this effect, defined by the supremum over all NEs $\overline{R}^{(n)}_{\mathrm{NE}}$ of the ratio between $\varphi(\overline{R}^{(n)}_{\mathrm{OPT}})$ and $\varphi(\overline{R}^{(n)}_{\mathrm{NE}})$, capturing the performance of the worst case equilibrium. We further consider the notion of *Price of Stability (PoS)* [2,3], defined by the infimum of the above ratio over all NEs, capturing the performance of the best case equilibrium.

Our Contribution. We study the rational choices of agents in an HIMA game, and analyze the performance of NE compared to the optimal performance. We focus on the case of homogeneous interferences, and show that when the utility of an agent is its success probability, then selfishness causes the system's performance to be up to an exponential factor away from the optimal performance. Specifically, we show that for any constant α, the price of anarchy as well as the price of stability are exponential in the number of agents, i.e., *any* equilibrium suffers an exponential degradation in performance. These results appear in Sec. 2.

We then turn to explore the effect of penalization, and to what extent does such an approach provide better system performance at a state of equilibrium. We show that there exists a penalty function which is proportional to the amount of aggressiveness demonstrated by an agent, such that for the case where the utility of an agent is the sum of its success probability and its penalty, then the price of stability with regards to the resulting *coordinated equilibria* can be made to drop to at most $e \approx 2.718$, thus demonstrating that an exponential improvement is possible compared to the uncoordinated case. We further show that for interferences which are not too large, namely, for $\alpha \leq 2/e \approx 0.735$, the price of anarchy is also bounded by e, thus ensuring that the degradation in performance due to the selfishness of the agents can be guaranteed to be made very small. These results mean that if we impose these penalties upon the agents, either in the form of payment for transmission to the network operator, or considering them as an intrinsic cost suffered by the agent due to transmission (e.g., due to power consumption), then the performance can be dramatically improved compared to the general case where the agent's utility is merely its success probability. These results are presented in Sec. 3. We note that our results for the homogeneous settings also extend to the finite horizon repeated HIMA game [4]. Due to space constraints, some of the proofs are omitted and may be found in [5].

Issues involving selfish behavior of agents in multiple access environments have received much attention in recent years. The slotted Aloha model in Markovian settings was studied in terms of stability conditions, and convergence to equilibrium (e.g., [6–9], and references therein). Additional works considered rate control games in wireless networks, (e.g., [10]), and other recent work [11–15] has also considered the role of introducing costs for transmissions, and pricing schemes, and their effect on the stability of the system. Other models of interferences in wireless networks in Markovian settings are discussed in [16–18].

2 General Nash Equilibria

In this section we present several analytical results as to the effect of selfishness on the performance of the network, in the theoretical case where interferences are homogeneous, i.e., for every i, j, $\alpha_{i,j} = \alpha$, for some system's parameter $\alpha \in (0, 1)$. We first consider the simple utility function $U_i(\overline{R}) = r_i(\overline{R})$, and show that in such a case, the system's performance can be very far from optimal. Specifically, we prove the following:

Theorem 1. *Given n stations, and any $k \in \{1, \dots, n-1\}$,*

1. *If $\alpha \in \left[\frac{1}{k+1}, \frac{1}{k}\right)$ then $\mathrm{PoA}^{(n)} = \mathrm{PoS}^{(n)} = \frac{k}{n(1-\alpha)^{n-k}}$.*
2. *If $\alpha \le \frac{1}{n}$ then $\mathrm{PoA}^{(n)} = 1$.*

Theorem 1 implies that for any constant α we have $\mathrm{PoA}^{(n)} = \mathrm{PoS}^{(n)} = 2^{\Omega(n)}$. In what follows we provide the necessary elements in order to prove the above theorem. The following lemma follows immediately from the definition of the utility function:

Lemma 1. *For utility functions $U_i(\overline{R}) = r_i(\overline{R})$, the only NE solution is obtained by the uniform profile $\overline{R}$ where every station i plays the strategy $R_i = 1$. The social welfare value of this NE is $n(1-\alpha)^{n-1}$.*

Lemma 1 implies in particular that since there is only one NE in these settings, then the price of stability equals the price of anarchy. In order to determine this value, we now turn to analyze the value of a profile which maximizes the social welfare. We first analyze the value of the social welfare function on the boundary of the profiles domain $[0, 1]^n$, and then turn to analyze the maximum value obtained in the interior of the domain.

Since φ is symmetric, any two integral profiles $\overline{R}, \overline{R}' \in \{0, 1\}^n$ having the same number of 1's, satisfy $\varphi(\overline{R}) = \varphi(\overline{R}')$. Let $B_k = (R_1, \dots, R_n)$ denote any profile with exactly k 1's. It therefore follows that the value in every extreme point where k stations play the 1-strategy and $n-k$ stations play the 0-strategy, is given by $v_k = \varphi(B_k)$ where

$$v_k = \sum_{i:R_i=1} R_i \prod_{j \neq i}(1-\alpha R_j) \quad + \sum_{i:R_i=0} R_i \prod_{j \neq i}(1-\alpha R_j) = k(1-\alpha)^{k-1}.$$

The following lemma characterizes the maximum value on the border of the profiles domain, and its dependence on α (proof omitted).

Lemma 2. *If* $\alpha \in [\frac{1}{k+1}, \frac{1}{k})$ *then* $\max_j v_j = v_k$.

Since clearly $\varphi(\overline{R}^{(n)}_{\mathrm{OPT}}) \geq v_k$ for all k and for all α, we therefore have $\varphi(\overline{R}^{(n)}_{\mathrm{OPT}}) \geq \max_k v_k$ for all α. In order to show that indeed $\varphi(\overline{R}^{(n)}_{\mathrm{OPT}}) = \max_k v_k$, we wish to show that the maximum of $\varphi(\cdot)$ is not obtained in the interior of the domain. The following lemma, whose proof is omitted, shows that there is only one possible extreme point $\overline{R}_0$ in the interior of the domain $(0,1)^n$, and characterizes the value of $\varphi(\overline{R}_0)$.

Lemma 3. *For any* n *and* $k \in \{1,\ldots,n-1\}$*, if* $\alpha \in [\frac{1}{k+1}, \frac{1}{k})$ *then the only possible extreme point of the social welfare function* $\varphi(\cdot)$ *in the interior of* $(0,1)^n$ *is* $\overline{R}_0 = (\frac{1}{\alpha n}, \ldots, \frac{1}{\alpha n})$*. Furthermore,* $\varphi(\overline{R}_0) \leq v_k$.

Combining lemmas 2 and 3 immediately implies the following corollary:

Corollary 1. *For any* n *and* $k \in \{1,\ldots,n-1\}$*, if* $\alpha \in [\frac{1}{k+1}, \frac{1}{k})$*, then* $\varphi(\overline{R}^{(n)}_{\mathrm{OPT}}) = v_k$.

Combining Lemma 1 which shows that there exists a single NE solution $\overline{R}^{(n)}_{\mathrm{NE}} = (1,\ldots,1)$ whose value is $\varphi(\overline{R}^{(n)}_{\mathrm{NE}}) = n(1-\alpha)^{n-1}$, along with Corollary 1, we can conclude the proof of Theorem 1.

As a consequence of Theorem 1, we restrict our attention in the following sections to the case where $\alpha \in (1/n, 1)$, since for $\alpha \leq 1/n$, the single NE of the HIMA game is indeed optimal. The following sections present a penalization scheme which enables the system to obtain a much better throughput, while still being at equilibrium.

3 Coordinated Nash Equilibria

In this section we introduce a penalty based scheme, where every station i incurs a penalty $p_i(\cdot)$ for transmission. We consider two types of penalties. The first type depends upon the choices of all the stations in the system, i.e., $p_i(\overline{R})$, while the second type only depends upon the choice of station i, i.e., $p_i(R_i)$. We refer to the former as an *exogenous penalty*, whereas the latter is referred to as an *endogenous penalty*. The general form of the utility function of station i is therefore $U_i(\overline{R}) = r_i - p_i$ (see [14] for a similar approach in the context of power control in cellular networks). We use the notion of coordinated NE and show that for both penalty functions, the selfishness of the stations does not result in more than a constant factor degradation in performance compared to the optimal performance. This should be contrasted with the results presented in the previous section showing that the price of stability for the uncoordinated case can be exponential in the number of stations.

We first show that there exists some $q_0 \in [0,1]$ such that the uniform profile $\overline{R}$ where $R_i = q_0$ for all i implies a mere constant degradation in performance compared to the optimal throughput possible. Note however that such a uniform profile need not be at NE. We then show that there exist penalty functions which cause such a uniform profile to be at NE. It therefore follows that by the use of appropriate penalties, selfishness can be tamed into providing a throughput that is at most a constant factor far from the optimal throughput.

3.1 The Power of Uniform Profiles

Given any $q \in [0,1]$, let $\overline{R}^q$ denote the uniform profile where $R_i = q$ for all i. Note that the social welfare value of $\overline{R}^q$ is given by the function $\psi(q) = nq(1-\alpha q)^{n-1}$. It is easy to verify that the value of q which maximizes $\psi(\cdot)$ is $q_0 = \frac{1}{\alpha n}$. It follows that the social welfare value of $\overline{R}^{q_0}$ is $\psi(q_0) = \frac{1}{\alpha}\left(1-\frac{1}{n}\right)^{n-1} \geq \frac{1}{e\alpha}$, where the inequality follows from the fact that $\left(1-\frac{1}{n}\right)^{n-1}$ is strictly monotone decreasing, and converges to $1/e$.

As we have seen, the optimal value of the social welfare function for $\alpha \in [\frac{1}{k+1}, \frac{1}{k})$[3] is obtained for a profile where k stations play the 1-strategy, and $n-k$ stations play the 0-strategy, resulting in a social welfare value of $\varphi(\overline{R}_{\mathrm{OPT}}^{(n)}) = k(1-\alpha)^{k-1} < k\left(1-\frac{1}{k}\right)^{k-1} \leq \frac{1}{\alpha}$.

It follows that $\frac{\varphi(\overline{R}_{\mathrm{OPT}}^{(n)})}{\varphi(\overline{R}^{q_0})} \leq e$. In the following sections we show that we can choose a penalty function such that the profile $\overline{R}^{q_0}$ is at NE.

3.2 Exogenous Penalties

Let $q \in [0,1]$, and consider the following utility function:

$$U_i^q(\overline{R}) = R_i \prod_{j \neq i}(1-\alpha R_j)(2q - R_i),$$

which can be cast as a utility function of the form $U_i^q(\overline{R}) = r_i(\overline{R}) - p_i^q(\overline{R})$, where the exogenous penalty is defined by $p_i^q(\overline{R}) = \prod_{j\neq i}(1-\alpha R_j)((1-2q)R_i + R_i^2)$. Assume all stations except for i play strategy q. It follows that

$$U_i^q(R) = R_i(1-\alpha q)^{n-1}(2q - R_i) = (1-\alpha q)^{n-1}(2qR_i - R_i^2). \tag{1}$$

By taking derivatives, we obtain that the maximum is obtained for $R_i = q$, i.e., the uniform profile $\overline{R}^q$ is at NE.

It therefore follows that the price of stability is at least $\max_q \frac{\varphi(\overline{R}_{\mathrm{OPT}}^{(n)})}{\varphi(\overline{R}^q)}$. In addition, since choosing $R_i = q$ is the best response of station i regardless of the strategy chosen by any station $j \neq i$, we can conclude that the uniform profile $\overline{R}^q$ is the only NE solution, hence the price of anarchy is the same as the price of stability.

Combining this result with the result presented in the previous section, for $q_0 = \frac{1}{\alpha n}$, we obtain the following theorem:

[3] Equivalently, $k < \frac{1}{\alpha} \leq k+1$.

Theorem 2. *For every station i there exists an exogenous penalty function $p_i(\overline{R})$ for which the price of anarchy, as well as the price of stability, are at most e.*

Although Theorem 2 guarantees that aggressiveness can be tamed by using exogenous penalties, this might not be completely satisfactory. Exogenous penalties incurred by a station might change even if the station does not change its strategy. This might not be considered a handicap if the penalties cannot increase if the station remains put, however in our case, other stations being less aggressive actually increases the penalty incurred by a station, even if this station does not change its strategy. We address this issue in the following section, and present an endogenous penalty scheme, in which the penalty imposed on a station depends solely on its strategy, where increased aggressiveness is matched by increased penalties.

3.3 Endogenous Penalties

In this section we use insights from Sec. 3.2, and discuss endogenous penalty functions where the penalty function of station i depends only on R_i. Specifically, given any $q \in [0, 1]$, we consider for every station i the utility function

$$U_i^q(\overline{R}) = r_i(\overline{R}) - p_i^q(R_i),$$

where $p_i^q(R_i) = (1 - \alpha q)^{n-1}((1 - 2q)R_i + R_i^2)$ is an endogenous penalty function.

Assume all stations but i play the strategy q. It follows that the utility function of station i is again defined by Eq. (1). This implies that the best strategy for station i to play is $R_i = q$, hence the uniform profile $\overline{R}^q$ is at NE. Similarly to Theorem 2, we thus obtain the following theorem:

Theorem 3. *For every station i there exists an endogenous penalty function $p_i(R_i)$ for which the overall price of stability is at most e.*

When considering the price of anarchy, the following theorem provides the conditions for which the uniform profile $\overline{R}^q$ is actually the *only* NE (proof omitted).

Theorem 4. *If $\alpha \leq \frac{2}{e}$, then for every station i there exists an endogenous penalty function $p_i(R_i)$ for which the overall price of anarchy is at most e.*

4 Conclusion and Open Questions

We present a generalization of the classic multiple access model, by considering spatial interferences parameter, modeling scenarios where different transmissions to succeed simultaneously. This new model captures the fact that collisions are a phenomenon experienced by the receiving end of transmissions, and it depends on the amount of interferences sensed by this receiver from the various simultaneous transmissions.

We show that for homogeneous interferences, if agents are selfish, then the system's performance at equilibrium can be up to an exponential factor far away from the optimal performance. We further introduce a penalty function to be cast on the agents, inducing a much better performance in an equilibrium, which is at most a factor of e away from the optimal performance.

Several interesting questions remain open. First, it would be interesting to obtain analytic guarantees as to the price of anarchy and the price of stability for non-homogeneous interferences. We believe that our results serve as a mere first step in understanding such interference-bound environments. Second, it is interesting to see if our model enables the design of better medium-access protocols, taking into account possible prior knowledge of inter-agents interferences.

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